ontario educational television

grade 8 geometric concepts

1966–67
September–December Series

Geometric Concepts

A SERIES OF 17 BROADCASTS
IN SUPPORT OF
GRADE 8 MATHEMATICS

Ontario Educational Television
Ontario Department of Education

Prepared and written by
IVAN ALLEN
Supervisor of Mathematics
North York Board of Education

Subject supervisor
G. A. KAYE
Assistant Superintendant
Curriculum Division
Department of Education

**For program times, refer to the ETV broadcast schedule*

Suggestions for Viewing Television in the Classroom

THE TELEVISION SET

1 Switch on the television set at least five minutes before the start of the program. Turn the volume control down and cover the picture by adjusting the doors of the set, or cover with drapery or other material. This will ensure a minimum of class interruption during the warm-up procedure.

2 Two minutes prior to telecast, make the necessary adjustments to the brightness and contrast controls to ensure picture clarity. Keep volume turned down.

3 Approximately twenty seconds prior to telecast time remove the screen cover and adjust the volume control. Try to avoid adjustments during the program telecast.

4 Window and other lighting reflections on the screen may occur if the television set is positioned at certain angles to light sources. This condition can be avoided by repositioning the television set or through the use of the cabinet, doors. If no doors, cardboard shields may be easily fashioned and fixed to the set.

ENVIRONMENTAL FACTORS

5 It is not necessary to black out the classroom. If lighting can be slightly dimmed by closing window drapery or switching off some lights, acceptable light level should result.

6 Tests should be made prior to the broadcast, to ensure that the maximum benefits of viewing and listening are available to each pupil. The seating arrangements will obviously vary with room shapes, type of furniture and number of pupils, but no pupil should be placed in a position that is greater than a 45° angle from a line drawn straight from the centre of the picture tube. Using a 23-inch screen, the minimum distance between pupil and picture should be approximately five feet, and maximum distance from picture should be approximately twenty feet. The television receiver should be raised to a height so that the centre of the picture tube is approximately 66 inches above floor level.

7 These approximate measurements indicate that a square or wide classroom shape is much better than a long narrow room unless of course, desks can be turned towards a long wall or aligned towards a corner.

CAUTION: The measurements shown above are approximate. They may not apply to all classrooms and are offered as a guide only. Long extension cords, antenna leads, and insecure structures for the elevation of the television set should be avoided. Pupils should be discouraged from assisting in setting up the television set or making any adjustments to it.

Grade 8 Mathematics
Geometric Concepts ETV Series

GENERAL INTRODUCTION

This series of 17 programs is based on the course of study for Grade 8 Mathematics (see Curriculum Bulletin 1.12B, unit six: Geometry and Measurement). The teacher's guides in this booklet supply essential information to the teacher for each of the first 13 programs (these will be telecast during the first school term). Similar guides for the remaining programs are being prepared and will be forwarded at a later date.

As there are 17 weekly programs in this series, and, as each program lends itself to approximately one follow-up class period, it is suggested that teachers plan their teaching of unit six to cover the 17 weeks of the telecast at two periods a week, rather than to plan for the five periods a week over a period of six to seven weeks as indicated in the curriculum bulletin. One period for the film and subsequent discussion, and approximately one period for the kind of follow-up activities suggested in the guides would assure the best use of these programs. Such planning would also tend to integrate the study of Geometry with the total Mathematics program. It is essential that the students be equipped with the materials as indicated in the guides for each program because the students in the classroom actually participate in activity during the telecast.

The programs are designed not only to stimulate student interest in and understanding of Geometry but also to present the teacher with some useful ideas concerning presentation of lesson material to students. The intuitive approach is evident as students discover and analyze situations and make conjectures which are often tested by measurement. Such experience should be beneficial to the student when confronted with mathematical problems in later years that require formal proof. Grade 7 work is reviewed briefly at the beginning of the series so that students with little or no background can get the full benefit of the programs.

Grade 8 Mathematics
Program 1
Geometry
The Science of Size and Shape

AIM

The first program in this series of telecasts on "Geometric Concepts" treats the subject as an intuitive study of shape and size. An attempt has been made to take the television camera outside the classroom to show many examples of geometric objects from the real world. In this way it is hoped that students will become more observant and aware of geometric structures in their environment and realize that Geometry plays an important part in their lives.

PREPARATION FOR TELECAST

It would be helpful if each classroom had a set of geometric models containing prisms, pyramids, cones, cylinders and spheres. The opportunity to see, handle, and discuss these before the program would benefit the student.

PROGRAM SUMMARY

The program begins by showing some of the shapes in the Universe, eventually focussing on our planet Earth. This is followed by film sequences showing some of nature's geometric handwork in plant and animal life.

The ingenuity of man is then recognized, as the camera portrays some very primitive, man-made structures followed by some famous and magnificent ones, which illustrate the deliberate use of geometric structure, particularly in the field of architecture. Some buildings are visited a second time, and in order to stress their basic three dimensional geometric shapes, fundamental models of these shapes are superimposed on them. As these models are analyzed, it is pointed out that their classifications depend upon the shapes of their faces.

Then, the faces are analysed and it is observed that they in turn depend upon their boundaries which are "simple closed curves". This topic "simple closed curves" receives some useful discussion.

The program goes on to suggest that three-dimensional figures can be envisaged in at least three ways. For example, the cube is shown first as a solid, then as a shell where just the surfaces are considered, and finally as a skeleton where just the edges are shown. Students are then encouraged to look for three dimensional shapes in their community environment and to try to classify them into sets of prisms, pyramids, cones, cylinders and spheres.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Students could be given ample opportunity to handle three-dimensional objects including prisms, pyramids, cones, cylinders and spheres and to discuss in their own words their respective attributes. In Program 3 these will be dealt with more fully after a discussion of plane figures in Program 2.

2 Students could be encouraged to look for examples of basic shapes in the world around them, and to bring examples or pictures of some of these to school. Such items not only add excitement to the class but also give an interesting décor to the classroom. Many of these objects can then be sub-classified as solids, shells or skeletons. For example,

a) a fence post is an example of a solid cylindrical shape.
b) a funnel is an example of a cone where the shell is of use.
c) a hydro tower may be considered a skeleton and is made up of prisms and pyramids.

BIBLIOGRAPHY:

Adventure in Geometry — Ravielli (Macmillan)

Grade 8 Mathematics
Program 2
Plane Figures in Geometry

AIM

The program begins by acknowledging the premise that we live in a three-dimensional concrete world. However, the abstract ideas of plane, point, and sets of points play a fundamental part in analysing and describing shapes within this world.

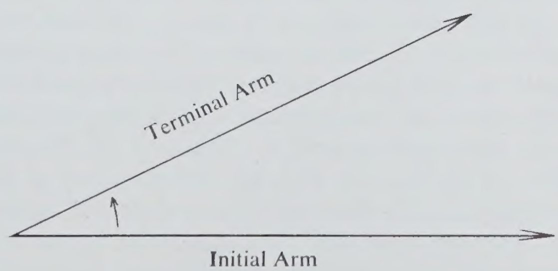
PREPARATION FOR TELECAST

In order to participate fully in this program, each student should have the following materials on his desk: pencil, ruler, protractor and workbook with the following table ruled on it.

Name of Polygon	*No. of Sides*	*Angle sum in right angles*
Triangle		
Quadrilateral		
Pentagon		

PROGRAM SUMMARY

The concepts of point, line, ray, segment, parallel and angle are reviewed in the early part of the program. The angle is discussed in two ways: first, as a plane figure formed by two rays with a common end point and, secondly, the measure of the angle is described as the amount of rotation of a ray about its end point, from an initial position to a final or terminal position. The following diagram indicates the rotation from the initial arm or position to the terminal arm or position.



Acute, right, obtuse and straight angles are then defined, followed by a description of the protractor and its use in measuring angles. Examples in constructing and measuring angles are given and the student audience is asked to participate.

Simple closed curves are mentioned again and the polygon is defined as a simple closed curve composed only of line segments. Then the triangle, quadrilateral, pentagon, hexagon, octagon and n-gon are defined as polygons with three, four, five, six, eight and "n" sides respectively. The program closes as on camera students attempt to find a method of determining the sum of the interior angles in any polygon. The student audience is asked to participate at the appropriate time by filling in the prepared table which was mentioned in the former section.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Students could be encouraged to discuss various definitions and points of interest or confusion in the telecast.
2 The table which was set up to find the sum of the angles in a polygon could be extended. Incidentally, the angle sum in an n-gon can be expressed as 2(n-2) right angles.
3 An exercise such as the following could be attempted and discussed:

a) How many lines can be drawn through one point?
b) How many line segments can be drawn to join two points?
c) How many segments can be drawn to join three (non-collinear) points?
d) How many segments can be drawn to join four (non-collinear) points A, B, C and D?
(There are six: AB, AC, AD, BC, BD, CD. Note the order in which I have named them.)
e) How many segments can be drawn to join the five points A, B, C, D and E?
(There are ten, namely AB, AC, AD, AE, BC, BD, BE, CD, CE, DE.)
f) How many segments can be drawn to join 10 points?
(By noting the pattern which is emerging in the former examples, we can get our answer from the series $9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$. Thus, 45 line segments could be drawn.)

Grade 8 Mathematics
Program 3
Solids and Surfaces

AIM

This program is designed to increase the student's awareness and knowledge of two and three dimensional figures.

PREPARATION FOR TELECAST

In order to participate fully in this program, each student should have a ruler, compasses and a notebook on his desk. It would be helpful if models of prisms, pyramids, cones, cylinders, and spheres were available so the students could handle and discuss them after the program.

If Bristol board and needles and thread are available, students can begin to do some curve stitching.

PROGRAM SUMMARY

The program opens by defining regular polygons as polygons with all sides and all angles equal.

Then triangles are classified according to sides (equilateral, isosceles, scalene) and according to the largest angles (acute angled, right angled and obtuse angled).

Quadrilaterals are also classified into sets of trapezoids, parallelograms, rectangles and squares.

The circle is defined as the set of all points equidistant from a given point; this is followed by descriptions of parts associated with the circle such as centre, minor and major arcs, radius and diameter.

The next portion of the program is devoted to description, definition, and discussion of three-dimensional figures. If a three-dimensional figure has plane faces which are all polygons, it is called a polyhedron. If the faces of the polyhedron are all congruent regular polygons, it is called a regular polyhedron. Only five of these exist. The tetrahedron is the simplest and has four equilateral triangles for faces. The hexahedron is commonly known as the cube and has six squares for faces. The octahedron and icosahedron have eight and twenty faces respectively, all being equilateral triangles. The dodecahedron has twelve regular pentagons for faces.

For a polyhedron to be a prism, the top and bottom, which are called bases, must be congruent and parallel polygons. The other faces or lateral surfaces must be parallelograms. Another common polyhedron is the pyramid. Its lateral surfaces are triangles with a common vertex while its base can be any polygon. Prisms and pyramids are classified and named according to their bases. Thus there are triangular prisms and triangular pyramids as well as hexagonal prisms and square pyramids.

Cones, cylinders, and spheres are discussed briefly.

A cone is then sliced in several ways and the circle, ellipse, and parabola are highlighted. Methods of drawing each of these curves are then illustrated, and the audience is encouraged to participate in the drawing of a parabola. The program closes as students practise the art of curve stitching.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Students could be encouraged to discuss the definition of the various polygons and solids, and thus review the essential features in each classification.
2 Plasticene models of solids could be made and then shapes noted as they are sliced in different ways.
3 Artistic designs could be created using geometrical instruments and also by means of curve stitching.

BIBLIOGRAPHY:

An Adventure in Geometry: Ravielli
Mathematics Enrichment: Ripley and Tait
Space Concepts Through Aestheometry: Anderson

Grade 8 Mathematics
Program 4
Geometry Involved In Model Construction

AIM

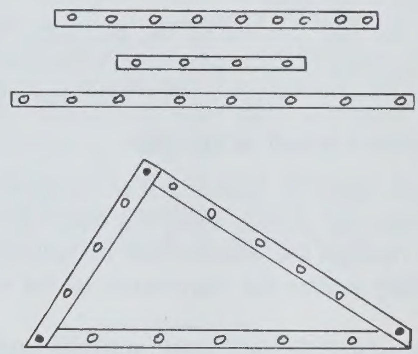
This program reinforces the idea that a knowledge of geometry is essential if man is to continue to design and build even the most basic structures in this world.

PREPARATION FOR TELECAST

Students should have pencils, rulers, and compasses on their desks so they can participate in the program.

The following materials would be useful in the follow-up activities after this program as well as several other programs to follow.

1 An assortment of flat bars of different lengths made from cardboard, wood, metal or plastic with holes drilled in them at regular intervals. Small bolts or paper fasteners are also necessary to make various polygons from these bars as shown below.



2 A package of drinking straws and several dozen pipe cleaners.

PROGRAM SUMMARY

Students gain a good deal of their early knowledge through experiences encountered during play. The program takes advantage of this aspect of student experience by having them illustrate methods of bricklaying with toy bricks as well as faulty and correct methods of tower building using model construction sets.

After moving to the classroom, students construct various polygons from wooden and plastic bars and illustrate that the triangle is the only rigid polygon. Three triangles with sides 6″, 7″, and 8″ are constructed and, by superimposing one on the other in turn, it is shown that they are identical in shape and thus we say they are congruent. However, when three quadrilaterals are constructed with sides 3″, 6″, 7″ and 9″, all three quadrilaterals are different in shape, because the sides are connected in different orders. The student audience is left with the question, "In how many different orders could these four sides be put together to form different quadrilaterals?" The answer to

the question is "three". The necessity of the triangle for rigidity in three dimensional skeletons is also illustrated as students make models using pipe cleaners and drinking straws.

A brief history of linear measure is discussed, and the following relationships are illustrated.

English System
1 yard = 3 feet = 36 inches
1 rod = 5½ yards
1 mile = 1760 yards
Metric System
10 millimetres (mm.) = 1 centimetre (cm.)
100 centimetres = 1 metre (m.)
1000 metres = 1 kilometre (km.)

It may be of interest for teachers to know that the yard is now officially defined in Canada, United States and United Kingdom as $\frac{9144}{10,000}$ of the international metre. The international metre is defined as "1,650,763.63 wavelengths in vacuum of the radiation corresponding to the transition between the levels of $2P_{10}$ and $5d_5$ of the krypton – 86 atom".

During the last portion of the program, the student audience is asked to participate. Instructions are given for the construction of triangles using ruler and compasses.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Students could be encouraged to construct two- and three-dimensional models using the materials suggested. It is only through the construction of such models that students fully realize the importance of the triangle for rigidity.

2 As a review of Grade 7 work, students could be given practice constructing triangles with rulers, compasses and protractors. The following examples will be discussed in Program 5, and it would be helpful to students if they had already done these and a few others like them.

a) Construct triangle XYZ when XY = 7.2 cm., angle XYZ = 48° and YZ = 9.4 cm.
b) Construct triangle DEF when angle DEF = 35°, EF = 10.4 cm., and angle DFE = 64°.
c) • Construct triangle MNO when MN = 3½ in., angle MNO = 30° and MO = 2¼ in.

BIBLIOGRAPHY:

The Romance of Weights and Measures: K. G. Irwin
The Story of Mathematics: Shaw and Fuge
The Wonderful World of Mathematics: Hogben
The Grant Golden Book of Mathematics: Adler

Grade 8 Mathematics
Program 5
To Construct an Angle Equal to a Given Angle

AIM

This program illustrates methods of angle construction in the real world and teaches students to construct angles equal to angles using only ruler and compasses.

PREPARATION FOR TELECAST

Students should have their solutions to questions 2a), 2b) and 2c) from the follow-up activities of Program 4 available as these will be discussed in the early part of this program. Ruler, compasses, and protractors are required for participation in the program.

PROGRAM SUMMARY

The program opens with a short historical discussion of the origin of geometry. We are told that the word itself originated over 2,000 years ago in Egypt, and was derived from two Greek words meaning "earth" and "measure". Apparently farmers along the Nile River were forced to abandon their farms for a period of time each spring due to floods. After the floods had subsided and they returned, the land formation had changed shape, and they had to re-survey their farms. In these surveys, certain rules were established and these became known as the rules of Geometry.

This first Geometry was very practical, and was mainly concerned with construction and measurement. However, when Greek civilization spread to Egypt, many of the Greek mathematicians began to study why the methods worked. Thus they created and organized a more theoretical or deductive type of Geometry.

The next portion of the program completes and discusses the constructions which were suggested by questions 2a), 2b) and 2c) in the follow-up activities of Program 4. It is noted that a unique triangle will be determined under the following conditions:

1 If the lengths of the three sides are given.
2 If the lengths of two sides and size of the contained angle are given.
3 If the lengths of one side and the size of two angles are given.

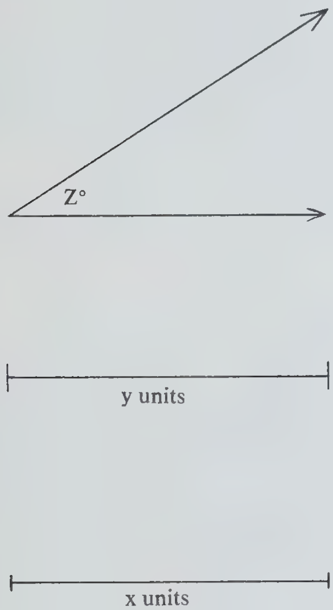
However, it is also pointed out that sometimes more than one correct triangle can be drawn from information which gives the lengths of two sides and an angle opposite one of them.

Students are then led to discover that (1) in an isosceles triangle the angles opposite the equal sides are equal, (2) if one angle of a triangle is greater than another, the side opposite the greater angle is greater than the side opposite the lesser angle.

Several practical examples of angle construction as used by carpenters are shown before a ruler-and-compasses method of constructing an angle equal to a given angle is developed. The remainder of the program allows students to practice this basic construction.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Discuss any part or parts of the program which created particular interest or difficulty. Students may be able to contribute other methods for angle constructions.
2 Students could be encouraged to bring practical construction tools and show them to the class.
3 Questions such as the following could be given to reinforce angle construction using ruler and compasses:
a) Construct a triangle ABC if AB = x units, BC = y units and angle ABC = Z°.



b) Construct any two angles and call them PQR and MNO. With ruler and compasses only, construct an angle equal to the sum of angles PQR and MNO.
c) Construct any triangle ABC. With ruler and compasses only, construct an angle equal to the sum of angle A, angle B, and angle C.

Grade 8 Mathematics
Program 6
To Bisect a Line Segment

AIM

This program is designed to help students discover how to bisect a given line segment and to gain insight into the process.

PREPARATION FOR TELECAST

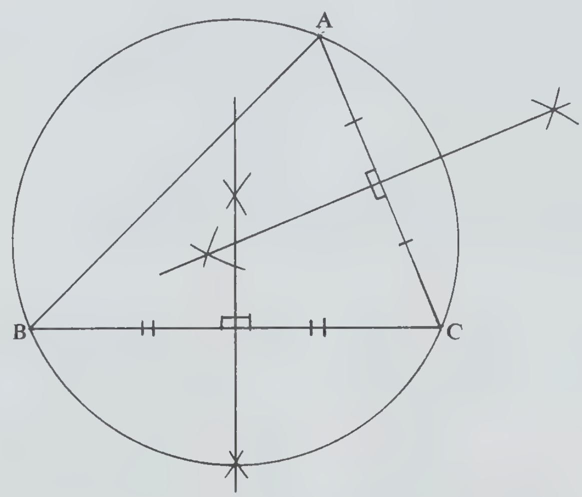
To participate in the program, students will need pencils, rulers, compasses and protractors. They should also have one or two loose sheets of unlined paper as well as their workbooks.

PROGRAM SUMMARY

Thales of Miletus (624 - 548 B.C.) is often called the Father of Geometry because he is the first known individual with whom mathematical discoveries are associated. Several amusing anecdotes about Thales are illustrated.

Students divide a rope into two, three, and four equal parts without using formal measurement. Then the student audience is asked to participate as a line on a page is bisected by paper folding. By examination of this method, students are led to discover a method of right bisecting a line segment using only ruler and compasses.

The audience participates in doing several examples which reinforce the idea of right bisection of a line segment, including an example where they construct the circumcircle of a triangle as shown below:



A short historical account of Euclid (about 365 B.C.) and his famous *Elements* is given. No work, except the Bible, has been more widely used or intensively studied. Over one thousand editions of Euclid's *Elements* have been printed since the invention of the printing press, and this work has dominated the teaching of geometry until recently. However, geometries other than "Euclidean Geometry" do exist, and one example involving the sum of the angles of a triangle on the surface of a sphere is illustrated.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Discuss the program regarding points of interest and difficulty. Note the difference between the "bisector" and the "right bisector" of a line segment. The "right bisector" must bisect the segment at right angles.

2 Questions such as the following could be given to reinforce the methods and concepts involved in this fundamental construction; namely, to bisect a given line segment.

a) Draw any line segment MN and divide it into four equal parts, using only ruler and compasses.
b) Draw a segment XY and then right bisect it. Mark points A, B, C, and D on the right bisector. With centres A, B, C, and D and radii AX, BX, CX, and DX, draw four circles. Do they each pass through Y? If so, why?
c) Construct the triangle ABC so that AB = 3 in., BC = 4 in., and AC = 2¾ in. Construct right bisectors to cut AB at X and AC at Y. Join XY and measure its length. What relationships do you see between XY and BC? (XY is parallel to BC and equal to one-half of it).
d) Draw a circle with centre O and radius 2½ in. Draw any chord AB in the circle and then right bisect this chord. Does this right bisector pass through any significant point? If a circle is given, could you find its centre by expanding the above method? (If two non-parallel chords are drawn, the right bisectors will intersect at the centre.)

BIBLIOGRAPHY:

The Story of Mathematics: Shaw and Fuge.
An Introduction to the History of Mathematics: Eves
Mathematics Enrichment: Ripley and Tait.

Grade 8 Mathematics
Program 7
To Bisect an Angle
To Construct a Line Perpendicular to a Line from a Point in the Line

AIM

This program is designed to show students some practical methods and needs for bisecting angles as well as to help them discover how to bisect an angle using only a ruler and compasses. The construction of a perpendicular to a line from a point in the line is introduced through the bisection of a straight angle.

PREPARATION FOR TELECAST

The usual geometric instruments are needed as the student audience is asked to participate in a good deal of the program. An assortment of the plastic, wood, metal, or cardboard bars as required in Program 4 are used during the program and would be useful in the follow-up activities.

PROGRAM SUMMARY

In earlier programs, we noticed that if polygons were made from plastic bars of specific lengths, the triangle was the only one that formed a rigid figure. The quadrilateral was treated lightly because of its flexibility. However, this program illustrates that many relationships remain the same in the quadrilateral, even though the shape changes. These relationships are analyzed and used to develop a method of bisecting an angle using only ruler and compasses.

Other methods of bisecting an angle are illustrated through paper folding, the use of a board, and the carpenter's square. Several examples are given which show a practical use for angle bisection.

The student audience is asked to participate in doing several examples involving the bisection of angles using only rulers and compasses. In one case, they bisect a straight angle and get two right angles. Further bisections allow them to construct angles of 45 degrees and 22½ degrees.

While it is extremely easy to bisect any given angle using only straight edge and compasses, it is impossible to trisect such an angle with these instruments. However, many devices have been devised that will trisect angles. The "tomahawk", which is shown in Figure 1, is one of them.

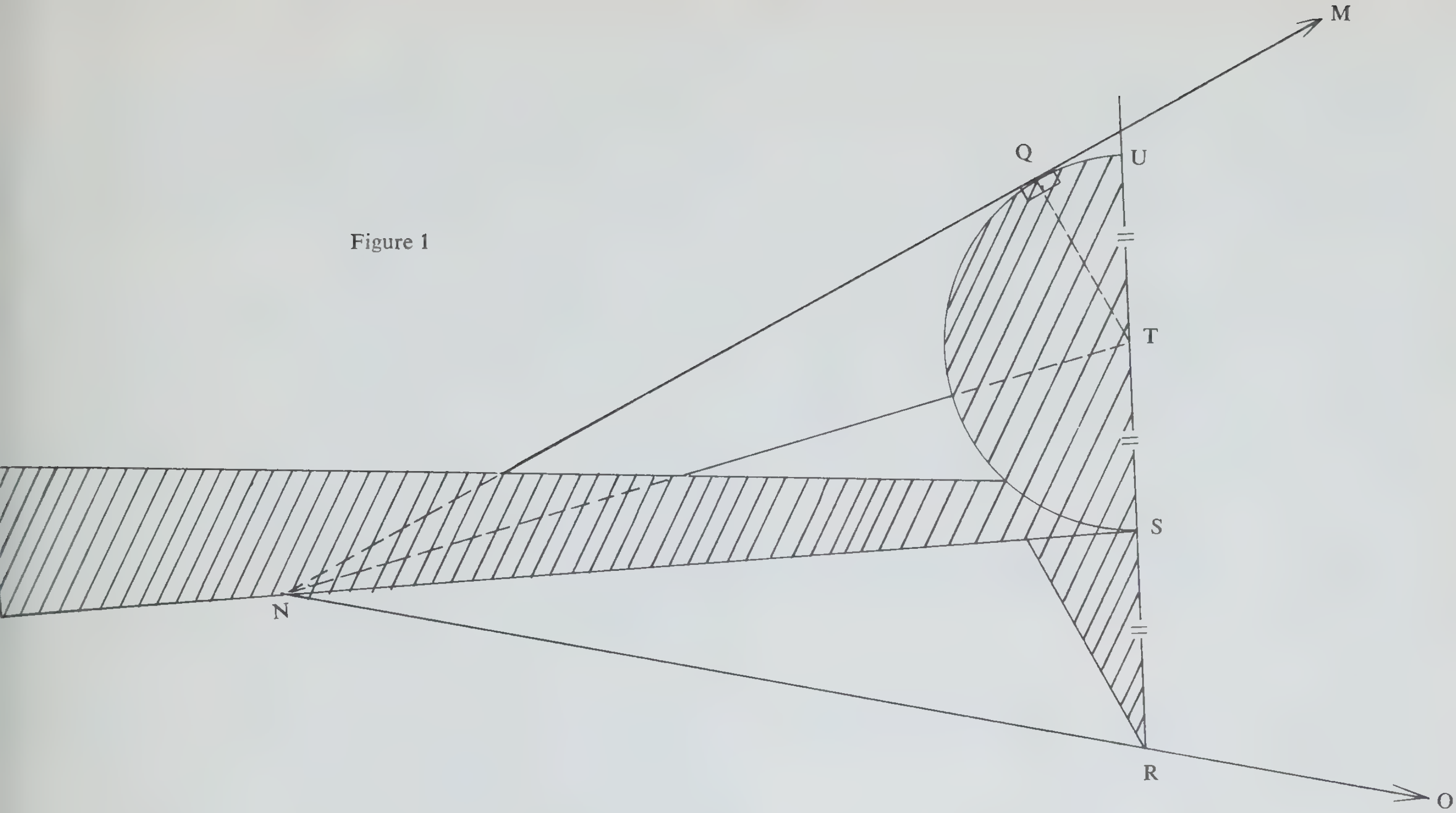


Figure 1

Because the "tomahawk" may be constructed while using only straight edge and compasses, the student audience is asked to construct one during the program.

In Figure 1 the shaded part represents a "tomahawk" which has been cut out of cardboard. It has been applied to angle MNO so that the angle is trisected by NS and NT. Note triangles RNS, SNT and TNQ. The program ends as students demonstrate the use of the "tomahawk" in trisecting angles.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Discuss points of interest and difficulty in the program and encourage students to look for instances where angle bisectors are used.

2 Students could be given the opportunity to devise and use simple tools, like those mentioned in the program, to bisect and trisect angles.

3 Questions such as the following could be assigned to reinforce methods and concepts in the two fundamental constructions which were discussed:

a) On the same diagram and using only ruler and compasses, construct and mark angles of 90 degrees, 45 degrees, 22½ degrees, 67½ degrees, 102½ degrees and 11¼ degrees.

b) Draw any acute angle ABC and then construct BD to bisect angle ABC. Let P and Q be any two points on BD. Using a set square construct PR and QS perpendicular to BC and meeting BC at R and S respectively. Draw circles with centres P and Q and radii PR and QS. What do you notice about these circles?

c) Construct two lines AB and CD to intersect at E. Construct EF and EG to bisect angles AED and AEC respectively. Measure angle FEG.

d) The mariner's compass has 32 directions. To draw such a compass construct a large circle centred on a page. Then construct a diameter to mark the East-West directions. Another diameter can be drawn perpendicular to this which shows the North-South directions. By continually bisecting angles at the centre 16 diameters can be drawn and thus the 32 equally spaced points of the compass will be indicated on the circle. Nine of these directions in order from North to East have the following names and abbreviations: North (N), North by East (N/E), North-Northeast (NNE), Northeast by North (NE/E), East-Northeast (ENE), East by North (E/N), East (E). Construct the compass and by extending the pattern indicated by these names, label the 32 directions on it.

BIBLIOGRAPHY:

An Introduction to the History of Mathematics: Eves

Grade 8 Mathematics
Program 8
To Draw a Line Perpendicular to a Line from a Point outside the Line

AIM

This program leads students towards the discovery of how to draw a line perpendicular to a line from a point outside the line, by reviewing and analyzing the former three fundamental constructions. Some work on parallel lines is also introduced so that students can discover relationships between alternate and corresponding angles.

PREPARATION FOR TELECAST

In order to participate in the program, the students will need rulers, compasses and protractors along with their workbooks.

PROGRAM SUMMARY

In the opening portion of the program, the student audience is asked to participate in the review of the following fundamental constructions:

1 To right bisect a line segment.
2 To bisect an angle.
3 To construct a line perpendicular to a line at a given point on the line.

After analyzing the concepts involved in these constructions, the student audience participates in the development of a method to construct a line perpendicular to a line from a point outside the line. This is followed by several examples which reinforce the method.

The next portion of the program is devoted to the study of parallel lines. When a line PS cuts two lines AB and CD, as shown in the diagram below, it is called a "transversal".

Pairs of angles such as AQP and CRQ or BQR and DRS are called "corresponding angles", whereas angles such as AQR and QRD or BQR and QRC are called "alternate angles". If the lines AB and CD are parallel as they are in this case, students discover through measurement that the angles in each pair of alternate or the corresponding angles are equal."

The program closes with an illustration showing how Eratosthenes (274 - 194 B.C.) measured the circumference of the earth. One of the key points in his solution applied the idea that "if a transversal meets two parallel lines, the corresponding angles are equal".

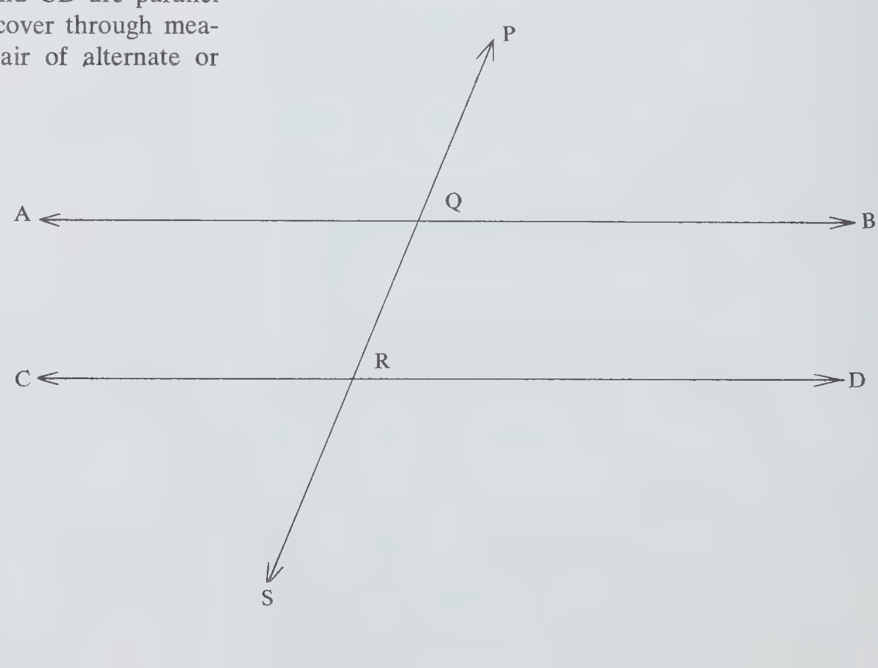
SUGGESTED FOLLOW-UP ACTIVITIES

1 Discuss the program and any difficulties arising from the fundamental constructions taken so far.
2 Corresponding angles and alternate angles could be reviewed so that each student is fully aware of their positions with respect to parallel lines and a transversal.
3 The story of how Eratosthenes measured the earth is well known but could lead to an interesting discussion. This is a case in which simple mathematics but great insight and understanding produced an amazing result.
4 Questions such as the following may be assigned to give further practice in the fundamental construction:

a) Construct a triangle ABC such that AB = 2½ in., BC = 3½ in., and AC = 2¾ in. Construct a line from A perpendicular to BC, meeting BC at D. AD is said to be an altitude of the triangle. Construct the other two altitudes from B and C. What do you notice about these three altitudes? (An altitude of a triangle is a line drawn from one vertex perpendicular to the opposite side.)
b) Construct triangle PQR such that PQ = PR = 10.4 cm. and QR = 8.0 cm. Draw a line from P perpendicular to QR meeting QR at S. Measure QS and RS.
c) Construct a triangle ABC such that AB = 12 cm., BC = 14 cm. and AC = 9 cm. Bisect angles A and C and let the bisectors meet at O. Construct OD perpendicular to BC meeting BC at D. With centre O and radius OD draw a circle. (This is called the in-scribed circle of a triangle.)

BIBLIOGRAPHY:

The Story of Mathematics: Shaw and Fuge.
Mathematics Enrichment: Ripley and Tait.



Grade 8 Mathematics
Program 9
To Construct a Line Parallel to a Given Line

AIM

This program is designed to help students realize the usefulness of parallel lines and to teach them to construct a line parallel to a given line through a given point outside the line.

PREPARATION FOR TELECAST

To participate in the program, students need the usual geometric instruments including ruler, compasses, and protractor. An assortment of plastic, wooden, or paper bars as described previously (see Program 4 guide) would be useful when discussing the program during the follow-up activities.

PROGRAM SUMMARY

The program opens showing students manipulating quadrilaterals made from plastic bars. It is noted that if the opposite sides of a quadrilateral are made equal, the sides remain parallel, no matter how it is flexed. By analyzing such a model, students are led to discover how to draw a line perpendicular to a given line through a given point outside the line, when using only ruler and compasses.

Several practical examples of the use of parallel lines and parallelograms are discussed. One of these shows the way a carpenter with a knowledge of geometry could divide a board 7⅜″ wide into three equal parts with a minimum of calculation. A ruler can be placed in each of the positions as shown in Figure 1. The 3-inch and 6-inch points are then marked. By joining these points as indicated by the dotted lines, the board is divided into three equal parts. The audience participates in doing a similar example where a page is divided into three equal columns.

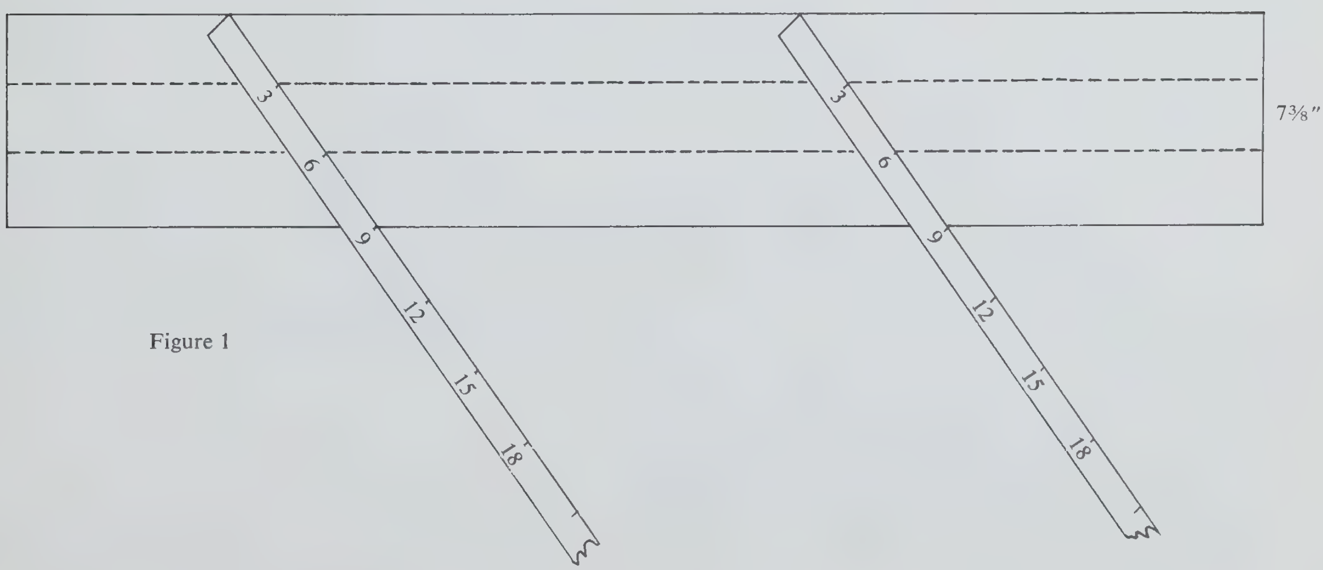
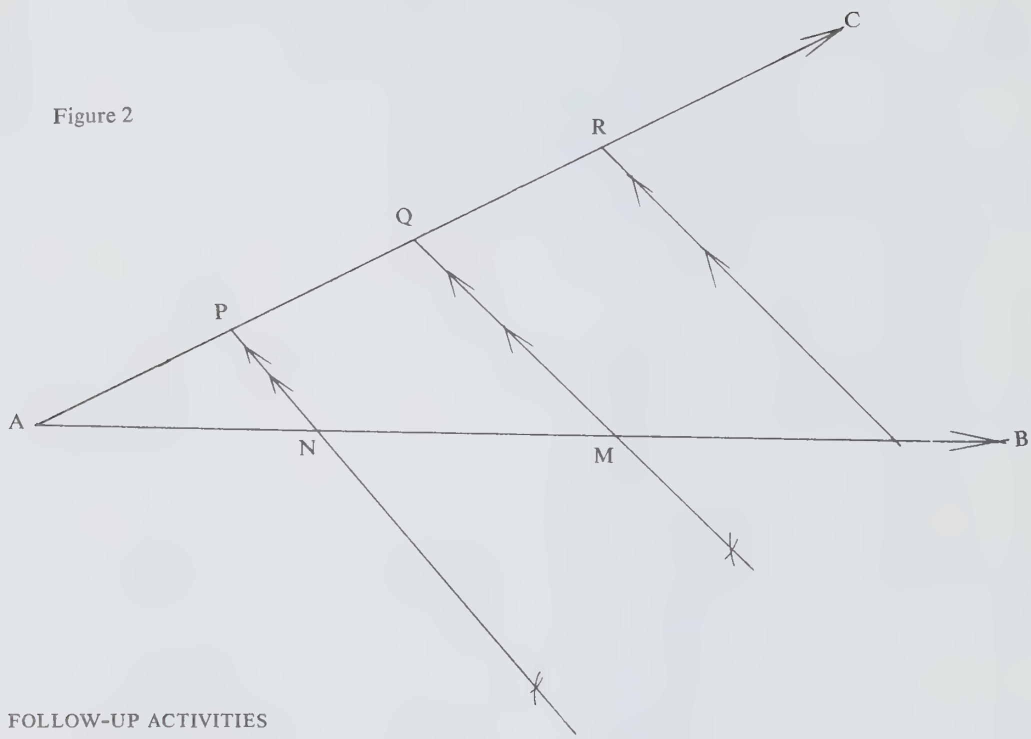


Figure 1

The program closes with an example illustrating how a line segment AB can be divided into three equal parts using only straight edge and compasses. A ray AC is drawn and then equal segments AP, PQ and QR are marked on it as shown in Figure 2. Then RB is joined and QM and PN are drawn parallel to RB. By measurement, it can be shown that AN = NM = MB.

Figure 2



SUGGESTED FOLLOW-UP ACTIVITIES

1 Discuss the different methods used for constructing parallel lines.

2 Students could construct models involving parallelograms from the plastic, wooden, or paper bars and then observe relationships as they are flexed.

3 Examples such as the following could give practice in the ruler-and-compasses method of constructing parallel lines.

a) Draw any triangle ABC and then construct a line through A parallel to BC. Find alternate angles in the diagram and measure them.

b) Construct a triangle ABC such that AB = 3 in., AC = 3½ in. and BC = 4 in. Mark D as the mid point of AB. Through D draw a line parallel to BC meeting AC at E. Measure AE, EC, DE and BC. Note any relationships which appear significant. (AE = EC and BE = ½BC.)

c) Construct a line segment YZ, about 6 in. long. Then draw a ray YX such that angle XYZ is an acute angle. With suitable radius, cut off five equal segments YA, AB, BC, CD, and DE on the ray YX. Join EZ. Now draw lines through A, B, C, and D parallel to EZ and meeting YZ at F, G, H and I respectively. Compare the lengths of the segments YF, FG, GH, HI and IZ.

Grade 8 Mathematics
Program 10
Perimeter and Circumference

AIM

This program is designed to reinforce the concepts of perimeter and circumference.

PREPARATION FOR TELECAST

To participate in the program, students need the usual geometric instruments and workbooks. For the follow-up activities, it would be helpful to have string and a flexible tape measure.

PROGRAM SUMMARY

The program begins with a review of closed curves and then introduces the concept of perimeter as the *length* of the curve. If the closed curve is a polygon the perimeter is found by taking the sum of the lengths of the sides. Shorter methods are indicated for special polygons but no formulas are given. Measuring the perimeter or circumference of a circle is somewhat more difficult, but students achieve this in different fashions.

Two experiments are shown which develop the relationship $C = \pi d$ where "c" and "d" represent the measures of the circumference and diameter of a circle. Different values for π are mentioned, and a short historical account of how Archimedes (287 - 212 B.C.) calculated a value for it is illustrated and described. It is noted that modern computers have calculated π to several thousand decimal places but in the problems which are discussed the approximate values of 3.14 and 22/7 are used.

(You may be interested to know that $\pi = 3.14159$,-26535,89793,23846, when it is calculated to 20 decimal places.) The program closes after the student audience has participated in the calculation of the perimeters of several figures.

SUGGESTED FOLLOW-UP ACTIVITIES

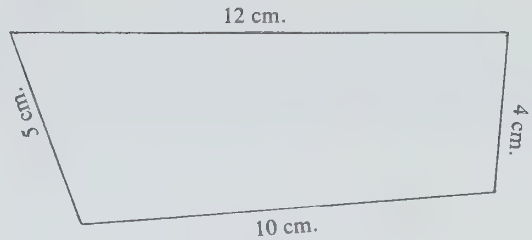
1 Discuss the concept of perimeter and have students give practical examples that would require the calculation of perimeter (e.g. the length of wallpaper border required for a room).

2 The class could find its own value for π through experiments similar to those used in the program, and the results could be shown in graphical form.

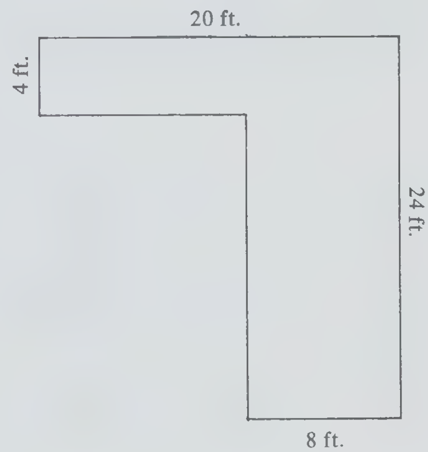
3 Students could be encouraged to find the actual perimeter of rooms, window frames, hallways, and the school yard by measurement and calculation.

4 Additional practice could be given by asking students to find the perimeters of figures similar to the following:

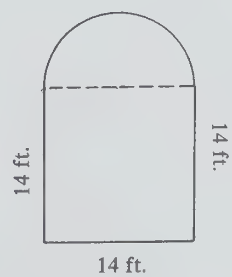
(a)



(b)



(c)



Grade 8 Mathematics
Program 11
Regions and Area

AIM

The aim of this program is to give students a sound understanding of the concept of area.

PREPARATION FOR TELECAST

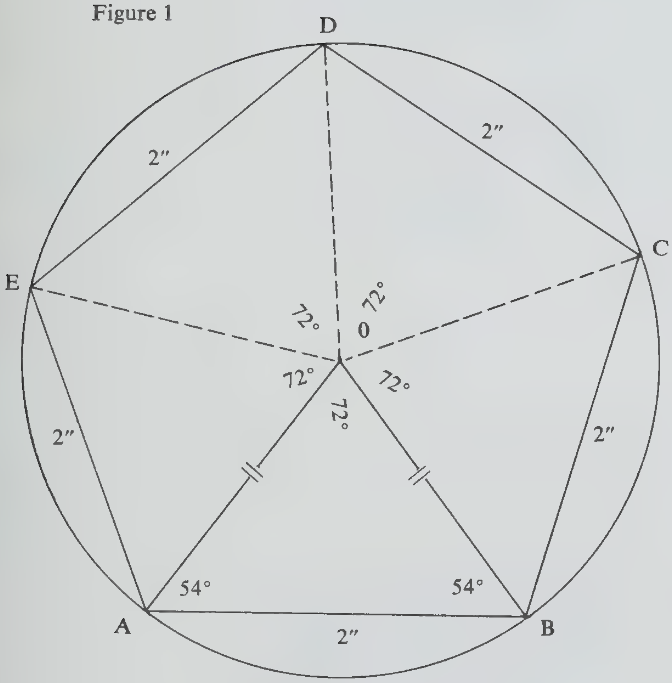
In order to participate in the program, the students need the usual geometric instruments and a workbook, with a table similar to the one below, ruled in it.

Relationships Between Perimeter and Area in Equilateral Triangles		
Measure of length of each side	*Measure of perimeter*	*Measure of Area*

For the follow-up activities, it would be convenient to have a supply of: a) graph paper with some marked in square inches and some in square centimetres; b) identical precut circles, triangles, squares, parallelograms, regular pentagons, regular hexagons, and regular octagons. (Because these are to be fitted together to form patterns, the sides of each polygon should be a constant length, e.g. 2 inches.)

The construction of a pentagon may give some difficulty but it (or any other polygon) can be constructed by using the following idea. The finished diagram of a pentagon will look like Figure 1. Each angle at the centre of the circle was calculated to be $360° \div 5 = 72°$. Then each base angle of the isosceles triangles was calculated to be $(180° - 72°) \div 2 = 54°$. In the actual construction, begin by drawing isosceles triangle AOB. Then with centre O and radius OA draw a circle. Points C, D and E may be found by constructing 72° angles at the centre or cutting off arcs equal to AB on the circumference.

Figure 1



PROGRAM SUMMARY

In order to establish the concept of the size of a region, students are asked to estimate how many sheets of construction paper it will take to cover a bulletin board. Each piece of paper is considered an arbitrary unit of size, and since 7½ sheets of paper are required, the area of the bulletin board is said to be 7½ units. The next example shows the same bulletin board covered with 18 unit triangles. Thus the area is said to be 18 units, as each unit is the size of one of these triangular pieces of construction paper. Either triangular or rectangular regions are shown to be suitable units for measuring area, but some time is spent experimenting to see if circles, octagons, hexagons or pentagons would also be suitable units.

After the concept of area is introduced, an experiment is carried out to show relationships between perimeter and area in equilateral triangles. Then examples are shown to indicate that a closed curve with a constant perimeter does not necessarily have a constant area and vice versa.

The square inch and the square centimetre are defined and then areas of irregular figures are determined by tracing their shapes on squared paper and counting the number of units they cover.

SUGGESTED FOLLOW-UP ACTIVITIES

1 The program could be discussed and students encouraged to carry out activities that would reinforce the concept of area. For example:

a) Compare the areas of different surfaces by covering them with identical precut polygons.
b) By using graph paper determine the area of irregular figures.
c) Compare the size of countries or lakes on a map by counting the number of equal square regions that each covers.

BIBLIOGRAPHY:

Mathematics for Primary Schools, Bulletin No. 1: British Schools Council.

Grade 8 Mathematics
Program 12
Units for Measurement of Area

AIM

This program attempts to give students an understanding of the different units which are commonly used to measure area and of some of the relationships between them.

PREPARATION FOR TELECAST

In order to participate in the program, students need the usual geometric instruments and workbooks.

PROGRAM SUMMARY

The formula (A = l x w) for finding the area of a rectangle is developed by observing the areas of rectangular regions marked on graph paper. Several units of square measure are introduced from both the British and Metric Systems and the following relationships are established.

British System	*Metric System*
1 sq. ft. = 144 sq. in.	1 sq. cm. = 100 sq. mm.
1 sq. yd. = 9 sq. ft.	1 sq. m. = 10,000 sq. cm.
1 sq. rd. = 30¼ sq. yd.	1 hectare = 10,000 sq. m.
1 acre = 160 sq. rd.	
1 sq. mi. = 640 acres	

Several problems are assigned which involve conversions between units within each system of measure.

Two methods of land survey are discussed briefly. In the new survey as used in Western Canada and Northern Ontario, farms are usually 160 acres and there are four of them in a square mile. In many older surveys concession roads are 1¼ miles apart, dividing the land into blocks containing 1,000 acres. These in turn are subdivided into ten farms of 100 acres each.

The program closes after the student audience has had an opportunity to calculate the area of several figures and to view sample solutions.

SUGGESTED FOLLOW-UP ACTIVITIES

1 Discuss the topics of interest and clarify points of difficulty.

2 Students could be encouraged to make charts indicating relationships between units. For example, a square with each side one foot long may be divided into 144 squares with each side one inch long. Also a scale diagram could indicate that one square rod is the same size as 30¼ square yards.

3 Students could be encouraged to find out how the land in their vicinity was surveyed. Many cities and towns were originally surveyed as townships and the main streets were formerly concessions or crossroads.

4 Students could be given the opportunity to find the area of convenient surfaces around the school, through measurement and calculation (e.g. the area of the classroom floor, the area of blackboard surface, the area of window surface, the area of the outside walls of the school, the area of the school yard).

5 Questions such as the following could be assigned to give additional practice in calculating area.

a) Find the area of Figure 1 and express the answer in square centimetres.

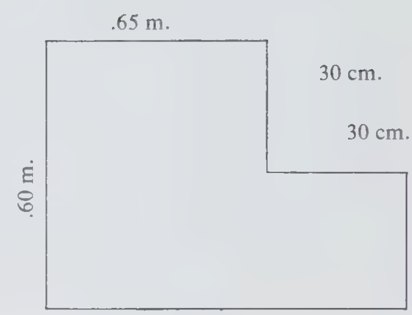


Figure 1

b) i) If a rectangular driveway is 15 feet wide and 63 feet long, find its area in square yards.
ii) Find the total cost to pave the driveway at the rate of $4.50 per square yard.

c) If a rectangular field is 40 rods by 24 rods, find its area in acres.

Grade 8 Mathematics
Program 13
Developing Formula for Calculating Area

AIM

This program is designed to help students understand and develop formulas for finding the area of parallelograms, triangles, and circles.

PREPARATION FOR TELECAST

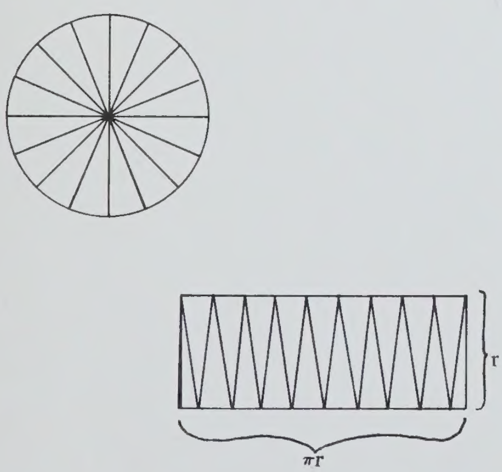
Students need the usual geometric instruments and workbooks to participate in the program. Construction paper or Bristol board would be useful in the follow-up activities.

PROGRAM SUMMARY

The program begins by reviewing the formula for the area of the rectangle ($A = lw$). The parallelogram is then considered, and it is cut and rearranged to form a rectangle of equal area. Then, the formula for the area of a parallelogram is established as $A = bh$ where "b" represents the length of one side of the parallelogram, and "h" represents the perpendicular distance from it to the opposite side.

A diagonal is drawn to a parallelogram and we see that this divides it into two congruent triangles. Thus the area of each triangle is half the area of the parallelogram. It is then shown that any two congruent triangles can form a parallelogram. Thus the formula "$A = \frac{1}{2}bh$" is developed for finding the area of a triangle where "b" represents the length of one side and "h" the length of the corresponding altitude.

The formula "$A = \pi r^2$" is developed for finding the area of the circle. A model with the circle divided into about 50 sectors is decomposed and rearranged to form a rectangle with length "πr" and width "r". A simplified version of this is shown in the diagram below.



Several examples are given to reinforce the concepts and methods involved in calculating area using formulas.

SUGGESTED FOLLOW-UP ACTIVITIES

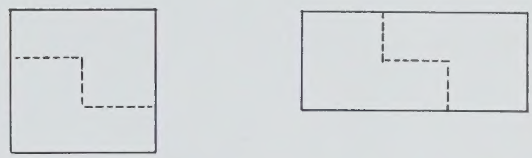
1 Discuss the program, and elaborate on points of interest, and clarify points of difficulty.

2 Students could be encouraged to cut and rearrange figures in different ways; for example,

i) Cut and rearrange a triangle to form a parallelogram.

ii) Cut and rearrange a rectangle to form a square. Can this be done with one cut? (see figures below).

Examples such as these will give insight into concepts behind formulas, and will also reinforce the idea of conservation of area.



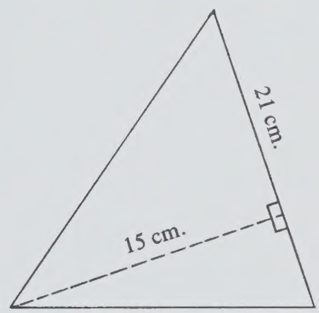
3 Questions such as the following could be given to allow practice in using the formulas.

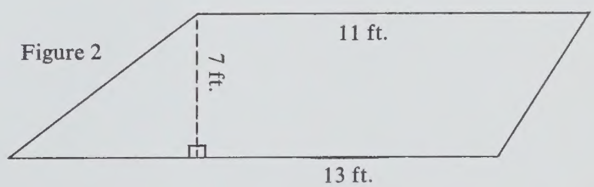
a) Find the area of the triangle in Figure 1.

b) By using formulas already developed, find the area of the trapezoid in Figure 2.

c) Calculate the area of the end of a house with dimensions as shown in Figure 3.

Figure 1





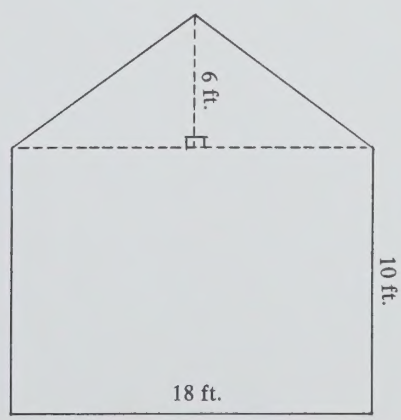


Figure 3